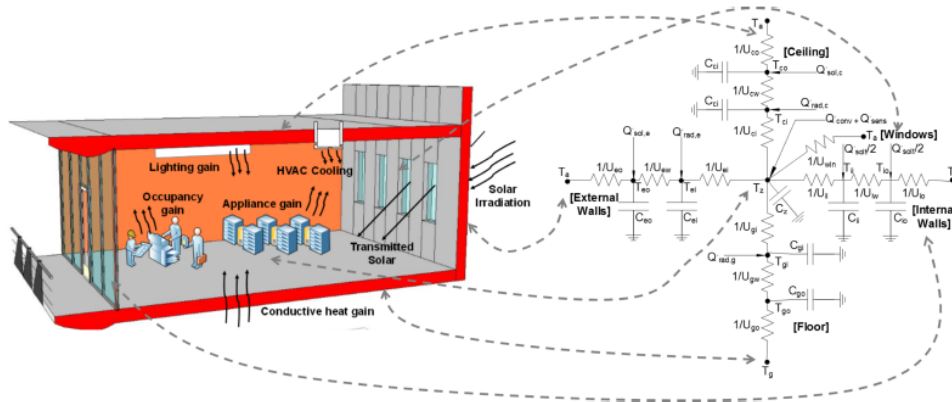

Parameter-Invariant Monitor Design for Cyber-Physical Systems: Part 2 – Design of Parameter-Invariant Monitors

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Recall CPS Applications



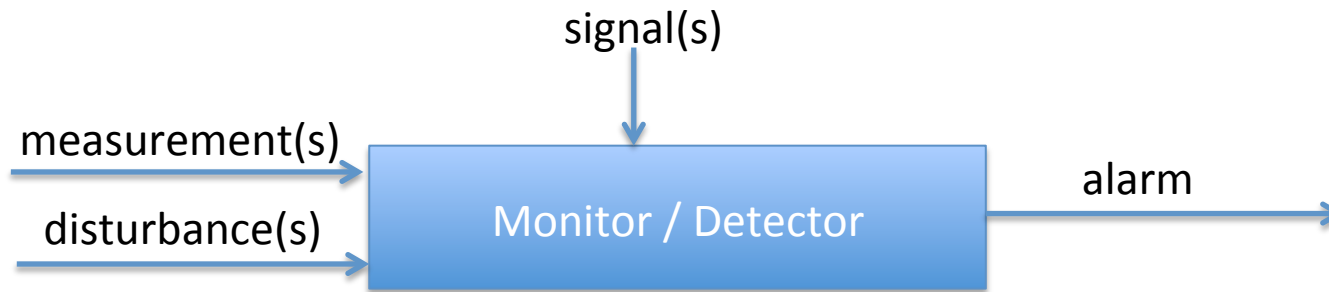
Recall The Monitor Design Problem

- Design a binary test between:
 - H_0 : null hypothesis
 - H_1 : event hypothesis

	H_0 is true	H_1 is true
test claims H_0	correct non-detection	missed detection
test claims H_1	false positive	true positive

- Performance constraints:
 - bound false positive rate
 - maximize true positive rate
- Last module covered different tests:
 - likelihood ratio test (LRT)
 - generalized likelihood ratio test (GLRT)
 - maximally invariant (MI) tests
 - parameter invariant (PAIN) tests
- This module covers the design of parameter-invariant monitors
 - modeling monitoring events
 - invariance to real-world dynamics

Monitor Architecture



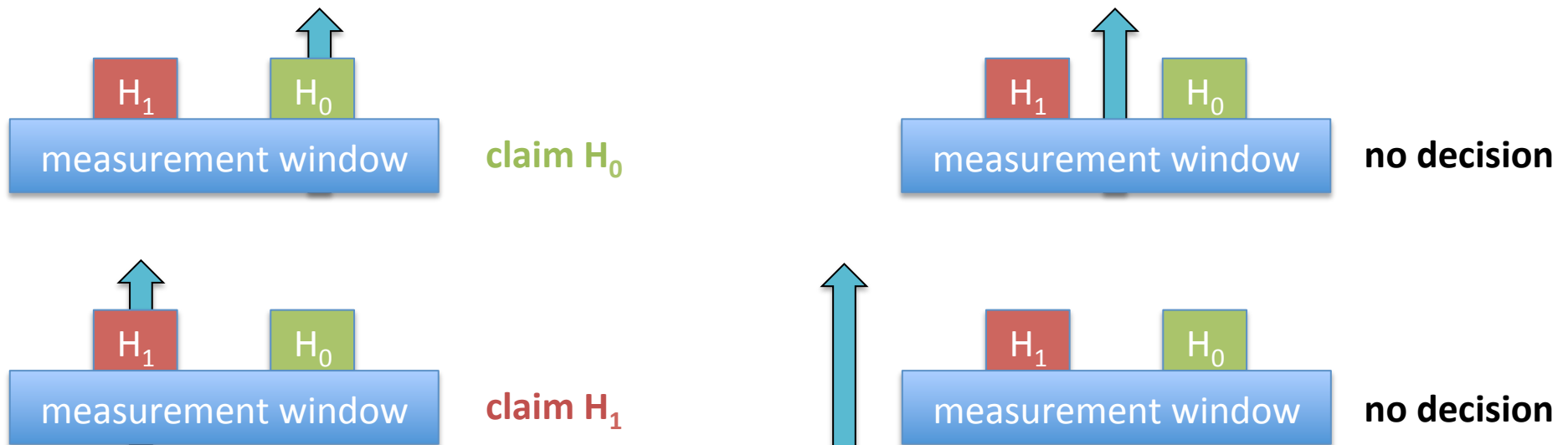
- measurement(s) – gathered from real-world sensor(s)
 - affected by monitoring event
- signal(s) – provided by sensors or domain expertise
 - unique for each hypothesis (ideally)
 - example: HVAC actuator fault detection using temperature measurements
 - H_0 : applied actuator voltage
 - H_1 : a constant
- disturbances(s) - provided by sensors or domain expertise
 - weakly correlated with monitoring event
 - potentially correlated with measurements
 - examples:
 - outdoor temperature – HVAC actuator fault detection using temperature measurements
 - insulin bolus – Diabetic meal detection using glucose measurements
 - respiratory rate – Hypoxia detection using expired CO_2

What is an “event”

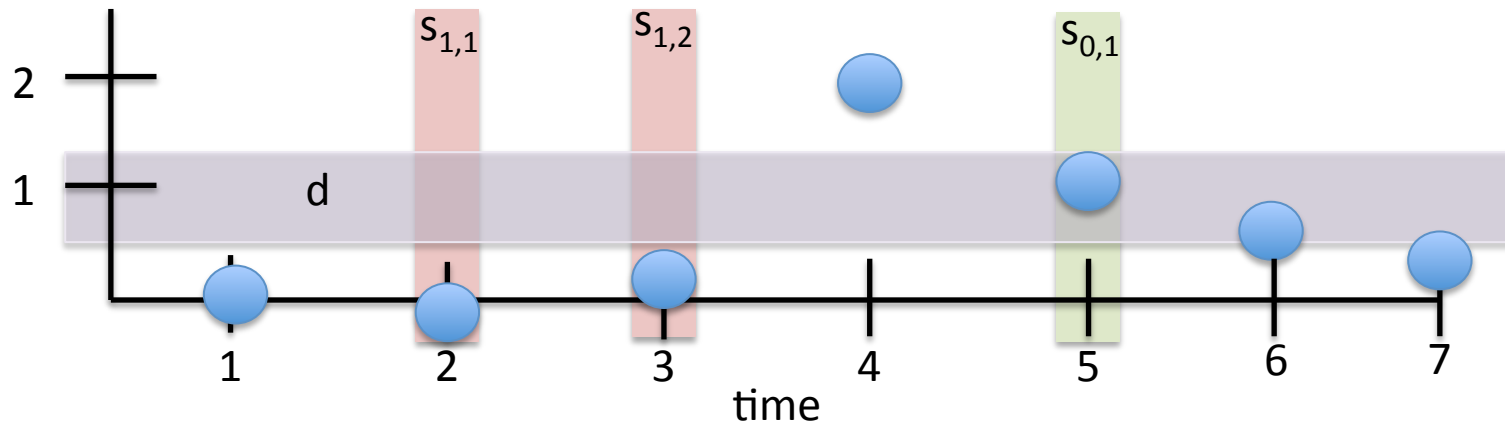
Modeling Monitoring Events

- Many ways to model “an event”: (not necessarily mutually exclusive)
 - presence/absence events
 - e.g. Does an input affect the output?
 - sequential events
 - e.g. In which time frame did the event occur?
 - dynamic events
 - e.g. Did the dynamics change?

rich paradigm



Sequential Monitoring Example



measurements

$$[y(1), y(2), y(3), y(4), y(5), y(6), y(7)]^T = [0.0, -0.1, 0.1, 1.9, 1.0, 0.6, 0.2]^T$$

disturbance

$$d(k) = d$$

null signal

$$s_{0,1}(k) = s_{0,1}\delta(k - 5)$$

event signals

$$s_{1,1}(k) = s_{1,1}\delta(k - 2)$$

$$s_{1,2}(k) = s_{1,2}\delta(k - 3)$$

$d, s_{0,1}, s_{1,1}, s_{1,2}$ are unknown

Finite-time Modeling of Physical Systems

- Consecutive temporal samples of inputs and measurements
 - inputs : test and/or nuisance
 - outputs (measurements) : dependent on inputs (and dynamics)
- Three classes of input-output dynamic models for physical systems
 - White box : model is known
 - Grey box : model is partially known
 - Black box : model is unknown
- What is the input-output relation?
 - linear / non-linear
 - static / dynamic
 - discrete / continuous dynamics
 - probabilistic / deterministic
 - etc.

Over a finite window of time,
(practically) all input-output relations
approximate one another with
bounded error

Linear Time Invariant Models

- Lets consider a linear time invariant model:
 - Easier to analyze than non-linear and time-varying models
 - well developed theory
 - many practical applications exist

$$y(k) = \sum_{n=1}^N a_n y(k-n) + \sum_{m=1}^M b_{m,n} u_m(k-n) + \sigma n(k) \quad (\text{general form})$$

known

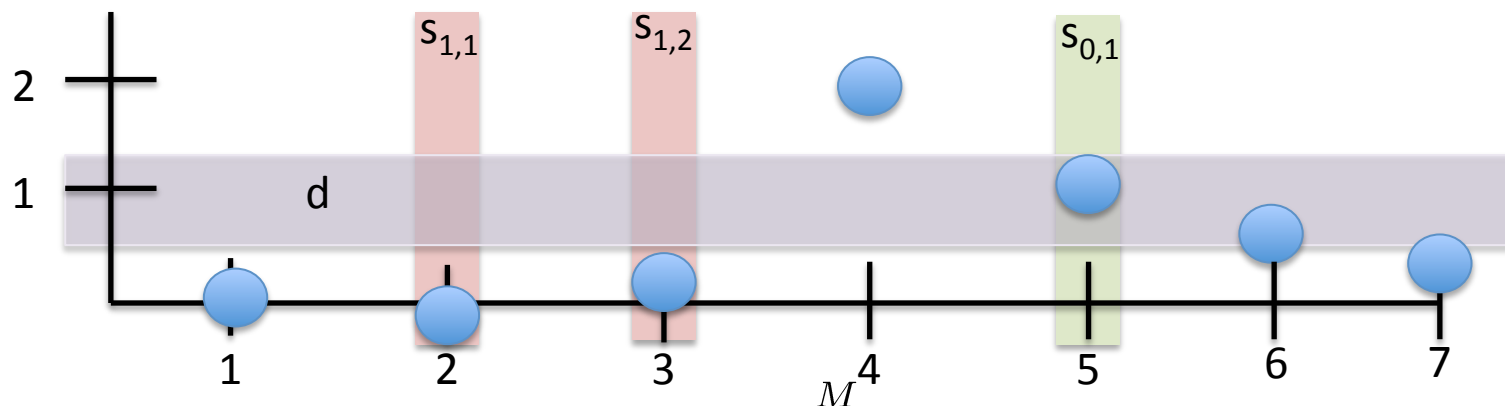
- model order: N
- number of inputs: M
- measurement: $y(k)$
- inputs: $u_m(k)$
 - test signals and disturbances
- noise mean: $E[n(k)] = 0$

unknown

- dynamics: $a_n, b_{m,n}$
- noise: $n(k)$
- noise scale: σ

let noise represent the model error

Sequential Monitoring Example



Model order: $N = 1 \rightarrow y(k) = a_1 y(k-1) + \sum_{m=1}^M b_{m,1} u_m(k-1) + \sigma n(k)$

Define inputs as concatenated test signals and disturbances: (4 inputs)

$$\begin{bmatrix} u_1(k) & u_2(k) & u_3(k) & u_4(k) \end{bmatrix} = \begin{bmatrix} s_{0,1}(k) & s_{1,1}(k) & s_{1,2}(k) & d(k) \end{bmatrix}$$

Resulting input-output relationships:

$$y(2) = a_1 y(1) + b_{4,1} + \sigma n(2)$$

$$y(3) = a_1 y(2) + b_{2,1} + b_{4,1} + \sigma n(3)$$

$$y(4) = a_1 y(3) + b_{3,1} + b_{4,1} + \sigma n(4)$$

$$y(5) = a_1 y(4) + b_{4,1} + \sigma n(5)$$

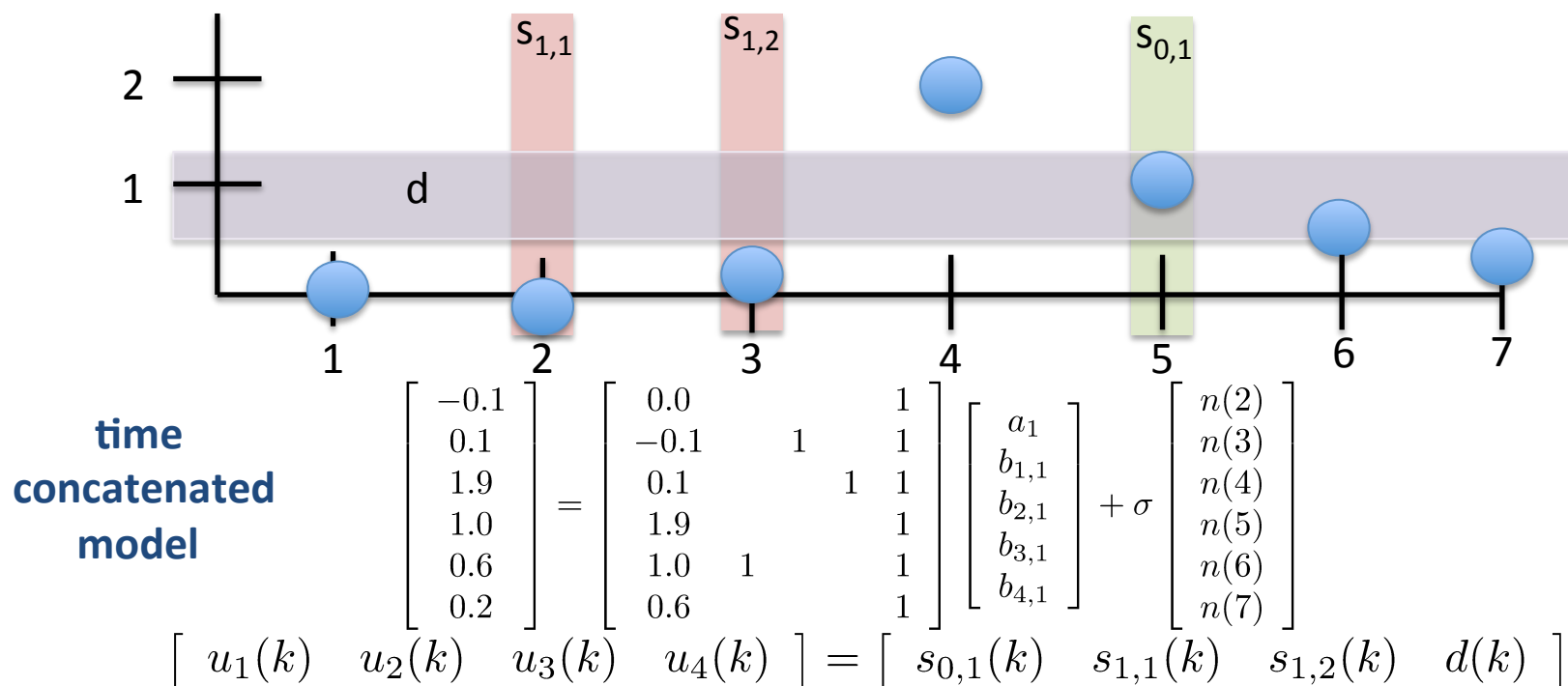
$$y(6) = a_1 y(5) + b_{1,1} + b_{4,1} + \sigma n(6)$$

$$y(7) = a_1 y(6) + b_{4,1} + \sigma n(7)$$

$$\begin{bmatrix} -0.1 \\ 0.1 \\ 1.9 \\ 1.0 \\ 0.6 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.0 & & & 1 \\ -0.1 & & 1 & \\ 0.1 & & & 1 \\ 1.9 & & 1 & \\ 1.0 & & & 1 \\ 0.6 & 1 & & \\ 0.6 & & & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_{1,1} \\ b_{2,1} \\ b_{3,1} \\ b_{4,1} \end{bmatrix} + \sigma \begin{bmatrix} n(2) \\ n(3) \\ n(4) \\ n(5) \\ n(6) \\ n(7) \end{bmatrix}$$

time concatenated model

Sequential Monitoring Example



null hypothesis $\rightarrow b_{2,1} = 0, b_{3,1} = 0$

$$\begin{bmatrix} -0.1 \\ 0.1 \\ 1.9 \\ 1.0 \\ 0.6 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.0 & 1 \\ -0.1 & 1 \\ 0.1 & 1 \\ 1.9 & 1 \\ 1.0 & 1 \\ 0.6 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_{1,1} \\ b_{4,1} \end{bmatrix} + \sigma \begin{bmatrix} n(2) \\ n(3) \\ n(4) \\ n(5) \\ n(6) \\ n(7) \end{bmatrix}$$

$$\mathcal{H}_0 : \mathbf{y} = \mathbf{H}_0 \theta_0 + \sigma \mathbf{n}$$

event hypothesis $\rightarrow b_{1,1} = 0$

$$\begin{bmatrix} -0.1 \\ 0.1 \\ 1.9 \\ 1.0 \\ 0.6 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.0 & 1 & 1 \\ -0.1 & 1 & 1 \\ 0.1 & 1 & 1 \\ 1.9 & 1 & 1 \\ 1.0 & 1 & 1 \\ 0.6 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_{2,1} \\ b_{3,1} \\ b_{4,1} \end{bmatrix} + \sigma \begin{bmatrix} n(2) \\ n(3) \\ n(4) \\ n(5) \\ n(6) \\ n(7) \end{bmatrix}$$

$$\mathcal{H}_1 : \mathbf{y} = \mathbf{H}_1 \theta_1 + \sigma \mathbf{n}$$

Linear Time Invariant Model for PAIN monitoring

- General LTI model form: $y(k) = \sum_{n=1}^N a_n y(k-n) + \sum_{m=1}^M b_{m,n} u_m(k-n) + \sigma n(k)$
- Window of T+N+1 measurements:

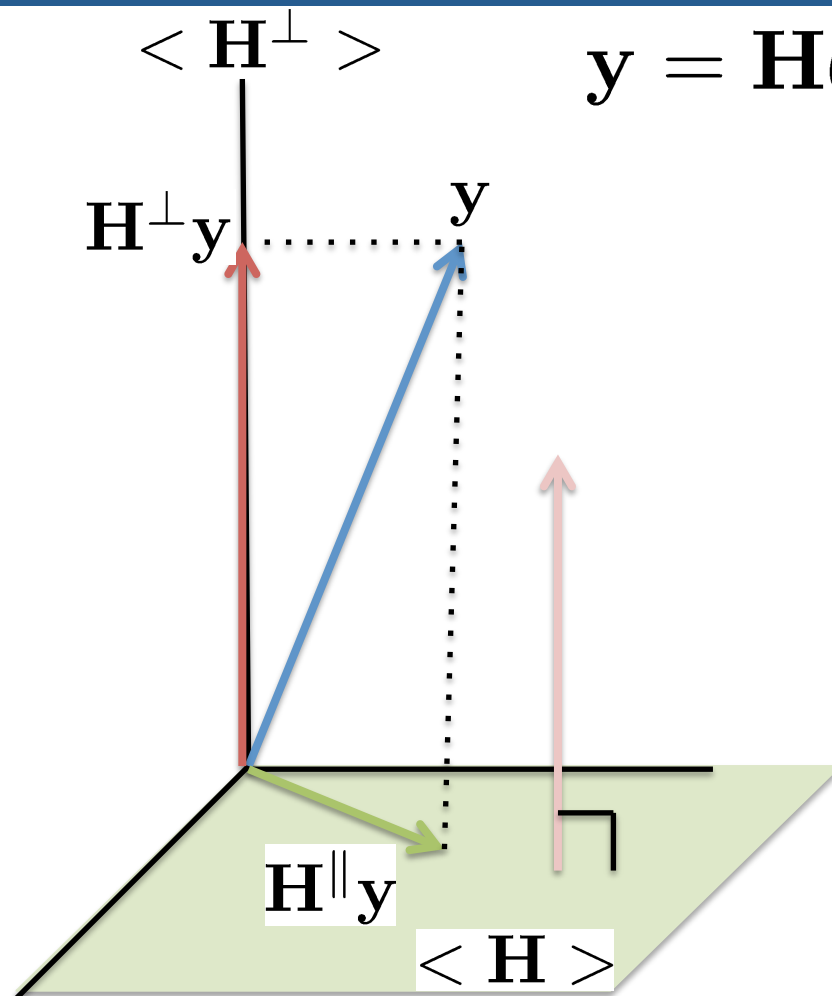
$$\begin{bmatrix} y(k-T) \\ \vdots \\ y(k-1) \\ y(k) \end{bmatrix} = \begin{bmatrix} y(k-T-1) & \dots & y(k-T-N) \\ \vdots & & \vdots \\ y(k-2) & \dots & y(k-N-1) \\ y(k-1) & \dots & y(k-N) \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} + \sum_{m=1}^M \begin{bmatrix} u_m(k-T-1) & \dots & u_m(k-T-N) \\ \vdots & & \vdots \\ u_m(k-2) & \dots & u_m(k-N-1) \\ u_m(k-1) & \dots & u_m(k-N) \end{bmatrix} \begin{bmatrix} b_{m,1} \\ \vdots \\ b_{m,N} \end{bmatrix} + \sigma \begin{bmatrix} n(k-T) \\ \vdots \\ n(k) \end{bmatrix}$$

$$\mathbf{y} = \mathbf{G}_0 \mathbf{a} + \sum_{m=1}^M \mathbf{G}_m \mathbf{b}_m + \sigma \mathbf{n}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \dots & \mathbf{G}_M \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_M \end{bmatrix} + \sigma \mathbf{n}$$

remove event signal from null hypothesis (and vice versa)

Nuisance Transformation



$$\mathbf{y} = \mathbf{H}\boldsymbol{\theta} + \sigma \mathbf{n}$$

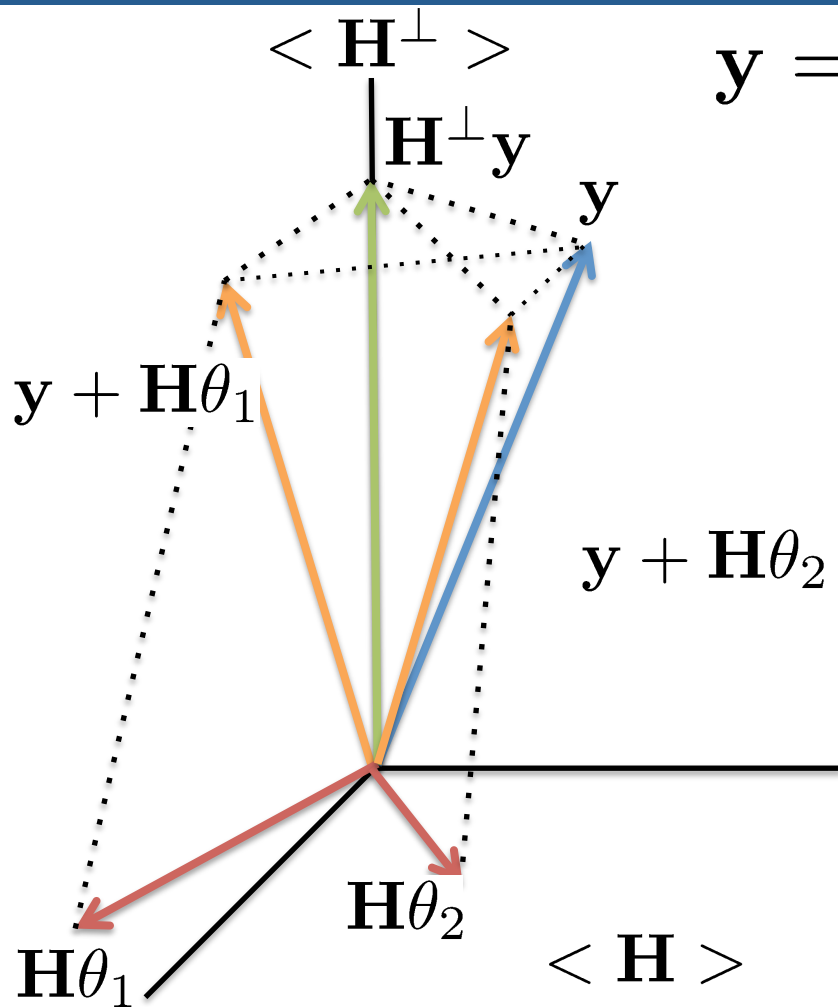
space of \mathbf{H} : $\langle \mathbf{H} \rangle$

projection onto $\langle \mathbf{H} \rangle$: \mathbf{H}^\parallel

nullspace \mathbf{H} : $\langle \mathbf{H}^\perp \rangle$

projection onto $\langle \mathbf{H}^\perp \rangle$: \mathbf{H}^\perp

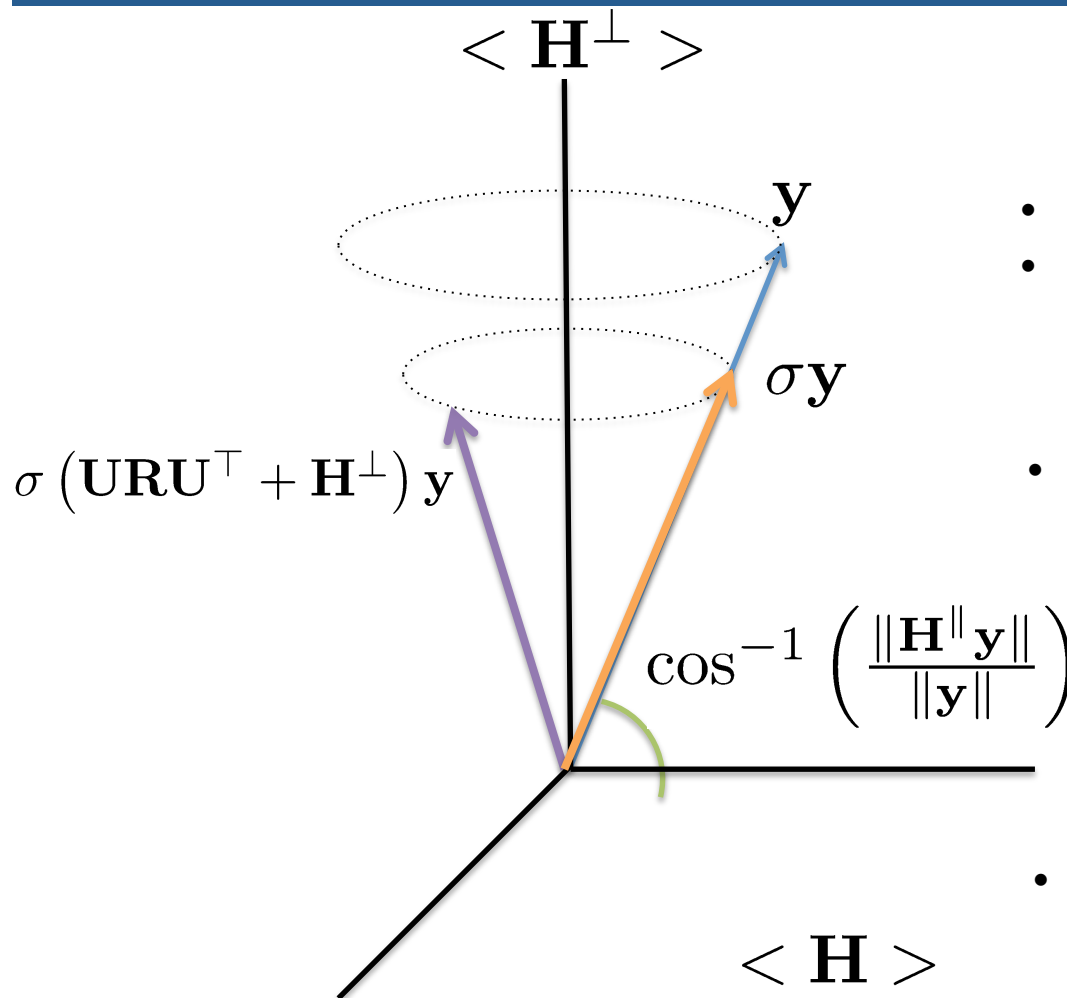
Nuisance Transformation : Translation



$$\mathbf{y} = \mathbf{H}\theta + \sigma \mathbf{n}$$

- translation induced by parameters corresponding to **nuisance inputs**
- maximally invariant statistic : $\mathbf{H}^\perp \mathbf{y}$
 - $\mathbf{H}^\perp \mathbf{H} = \mathbf{0}$

Nuisance Transformation : Scale and Rotation



$$\mathbf{y} = \mathbf{H}\theta + \sigma\mathbf{n}$$

- scale induced by unknown model error
- rotation induced when events can happen at multiple time steps
 - “don’t care when, just if it happens”

- mathematical relationships:

$$\mathbf{U}\mathbf{U}^\top = \mathbf{H}^\parallel$$

$$\mathbf{R}\mathbf{R}^\top = \mathbf{I}$$

- maximally invariant statistic : $\frac{\|\mathbf{H}\|\mathbf{y}\|}{\|\mathbf{y}\|}$

Parameter Invariant Test

hypothesis testing problem

$$\mathcal{H}_0 : y = \mathbf{H}_0 \theta_0 + \sigma \mathbf{n}$$

$$\mathcal{H}_1 : y = \mathbf{H}_1 \theta_1 + \sigma \mathbf{n}$$

- maximally invariant statistics:
 - translation : $\mathbf{H}^\perp \mathbf{y}$
 - rotation and scale : $\frac{\|\mathbf{H}^\perp \mathbf{y}\|}{\|\mathbf{y}\|}$

“composition of maximally invariant statistics is maximally invariant”

- Parameter Invariant statistics:

- constant false positive rate:**

$$\hat{\mathbf{y}}_0 = \mathbf{H}_0^\perp \mathbf{y}, \quad \hat{\mathbf{H}}_0 = \mathbf{H}_0^\perp \mathbf{H}_1, \quad t_0 = \frac{\|\hat{\mathbf{H}}_0^\perp \hat{\mathbf{y}}_0\|}{\|\hat{\mathbf{y}}_0\|}$$

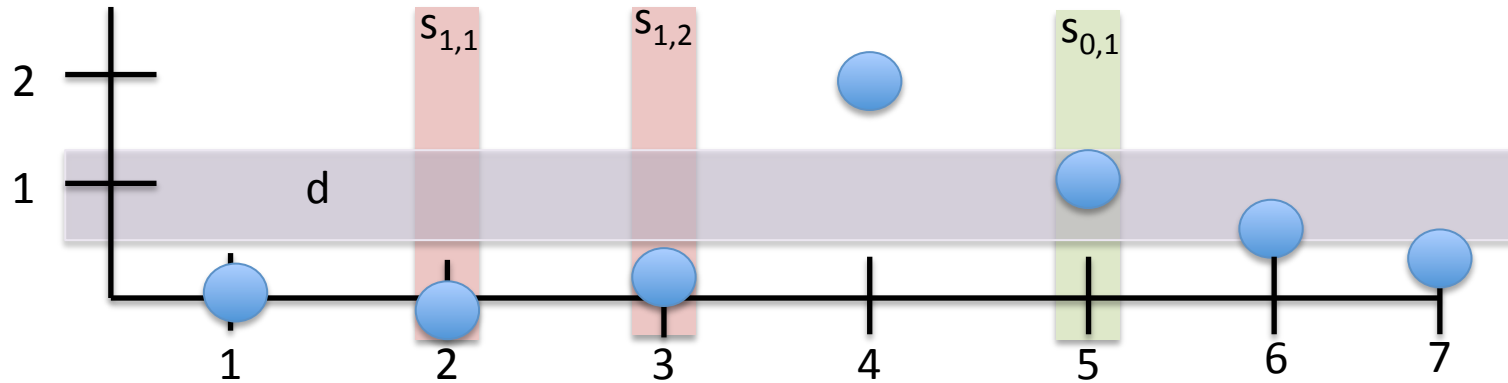
- constant true positive rate:**

$$\hat{\mathbf{y}}_1 = \mathbf{H}_1^\perp \mathbf{y}, \quad \hat{\mathbf{H}}_1 = \mathbf{H}_1^\perp \mathbf{H}_0, \quad t_1 = \frac{\|\hat{\mathbf{H}}_1^\perp \hat{\mathbf{y}}_1\|}{\|\hat{\mathbf{y}}_1\|}$$

- 2-sided parameter invariant test:

$$\phi(t_0, t_1) = \begin{cases} 1, & t_0 > \eta_0 \wedge t_1 \leq \eta_1 \\ 0, & t_0 \leq \eta_0 \wedge t_1 > \eta_1 \\ ?, & \text{else} \end{cases}$$

Sequential Monitoring Example



$$\mathcal{H}_0 : y = \mathbf{H}_0 \theta_0 + \sigma \mathbf{n}$$

$$\mathcal{H}_1 : y = \mathbf{H}_1 \theta_1 + \sigma \mathbf{n}$$

$$y = \begin{bmatrix} -0.1 \\ 0.1 \\ 1.9 \\ 1.0 \\ 0.6 \\ 0.2 \end{bmatrix}$$

$$\mathbf{H}_0 = \begin{bmatrix} 0.0 & 1 \\ -0.1 & 1 \\ 0.1 & 1 \\ 1.9 & 1 \\ 1.0 & 1 \\ 0.6 & 1 \end{bmatrix}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0.0 & 1 & 1 \\ -0.1 & 1 & 1 \\ 0.1 & 1 & 1 \\ 1.9 & 1 & 1 \\ 1.0 & 1 & 1 \\ 0.6 & 1 & 1 \end{bmatrix}$$

matlab code

$$\hat{\mathbf{H}}_0 = \text{null}(\mathbf{H}'_0)' * \mathbf{H}_1 \approx \begin{bmatrix} 0 & 0.59 & 0.27 & 0 \\ 0 & -0.57 & 0.77 & 0 \\ 0 & 0.04 & 0.29 & 0 \end{bmatrix}$$

$$\hat{\mathbf{y}}_0 = \text{null}(\mathbf{H}'_0)' * \mathbf{y} \approx \begin{bmatrix} 0.68 \\ 1.34 \\ -0.59 \end{bmatrix}$$

$$t_0 = \text{norm}(\text{orth}(\hat{\mathbf{H}}_0)' * \hat{\mathbf{y}}_0) / \text{norm}(\hat{\mathbf{y}}_0) \approx 0.9997 \quad t_1 \approx 0.9515$$

$\cos^{-1}(t_0) \sim 1$ degree
 $\cos^{-1}(t_1) \sim 18$ degrees

Module Summary

- Given any hypothesis testing problem of the form:

$$\mathcal{H}_0 : \mathbf{y} = \mathbf{H}_0\theta_0 + \sigma\mathbf{n}$$

$$\mathcal{H}_1 : \mathbf{y} = \mathbf{H}_1\theta_1 + \sigma\mathbf{n}$$

- the 2-sided PAIN test presented herein generates a robust monitor.
 - sequential events is just 1 approach that results in this form
- The following module explores the application of the 2-sided PAIN test herein
 - meal-detection for type I diabetics
 - early detection of hypoxia in infants
 - heating, ventilation, and air conditioning (HVAC) actuator fault detection