Robust Medical Monitor Design: Supplemental Slides

James Weimer, Oleg Sokolsky, Insup Lee

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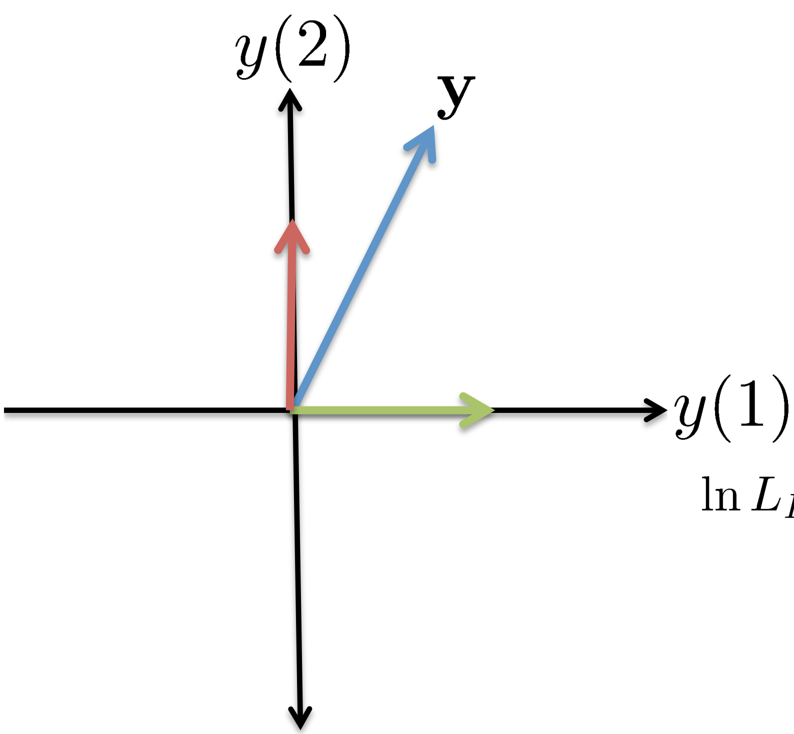
PRECISE

Example 1 : LRT

$$\mathcal{H}_0 : (\theta_1, \theta_2) \in \{1\} \times \{0\}$$
$$\mathcal{H}_1 : (\theta_1, \theta_2) \in \{0\} \times \{1\}$$



apply the likelihood ratio test

distribution of measurements

$$f(\mathbf{y}|\theta) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\sum_{k=1}^{2}(y(k)-\theta_k)^2\right\}$$

log-likelihood of parameters

$$\ln L(\theta; \mathbf{y}) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\sum_{k=1}^{2}(y(k)-\theta_k)^2$$

log-likelihood ratio

$$\begin{aligned}\ln L_R(\mathbf{y}) &= \ln L\left(\Theta_1; \mathbf{y}\right) - \ln L\left(\Theta_0; \mathbf{y}\right)\\ &= \frac{1}{2}\left(\sum_{k=1}^{2}(y(k)-\Theta_{0,k})^2 - \sum_{k=1}^{2}(y(k)-\Theta_{1,k})^2\right)\\ &= \frac{1}{2}\left((y(1)-1)^2 + y(2)^2 - y(1)^2 - (y(2)-1)^2\right)\\ &= y(2) - y(1)\end{aligned}$$

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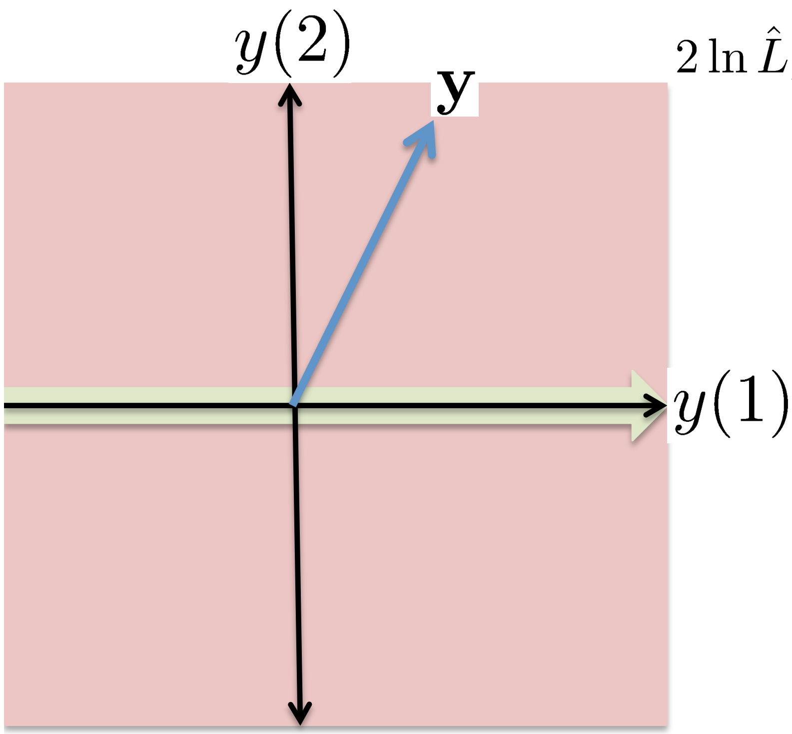
Example 2 : GLRT

$$\mathcal{H}_0 : (\theta_1, \theta_2) \in \mathbb{R} \times \{0\}$$
$$\mathcal{H}_1 : (\theta_1, \theta_2) \in \mathbb{R} \times \mathbb{R}$$

apply the generalized likelihood ratio test



generalized log-likelihood ratio

$$2 \ln \hat{L}_R(\mathbf{y}) = \max_{\hat{\theta}_1 \in \Theta_1} 2 \ln L\left(\hat{\theta}_1; \mathbf{y}\right) - \max_{\hat{\theta}_0 \in \Theta_0} 2 \ln L\left(\hat{\theta}_0; \mathbf{y}\right)$$
$$= \min_{\hat{\theta}_0 \in \mathbb{R} \times \{0\}} \sum_{k=1}^{2} (y(k) - \hat{\theta}_{0,k})^2$$
$$- \min_{\hat{\theta}_1 \in \mathbb{R} \times \mathbb{R}} \sum_{k=1}^{2} (y(k) - \hat{\theta}_{1,k})^2$$
$$= y(2)^2$$

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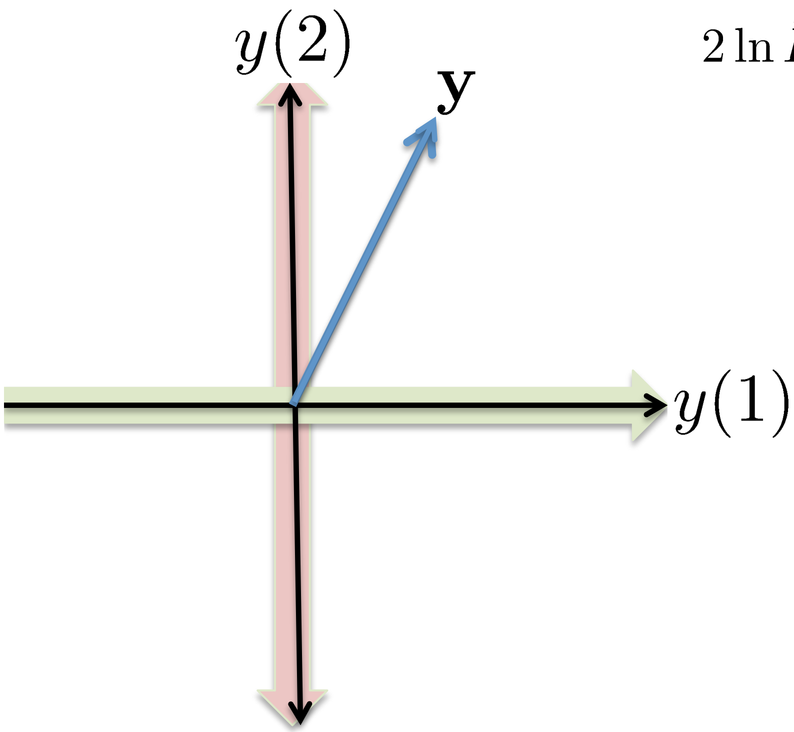
Example 3 : GLRT

$$\mathcal{H}_0 : (\theta_1, \theta_2) \in \mathbb{R} \times \{0\}$$
$$\mathcal{H}_1 : (\theta_1, \theta_2) \in \{0\} \times \mathbb{R}$$

apply the generalized likelihood ratio test



generalized log-likelihood ratio

$$
\begin{aligned}
2\ln \hat{L}_R(\mathbf{y}) &= \max_{\hat{\theta}_1 \in \Theta_1} 2\ln L\left(\hat{\theta}_1; \mathbf{y}\right) - \max_{\hat{\theta}_0 \in \Theta_0} 2\ln L\left(\hat{\theta}_0; \mathbf{y}\right) \\
&= \min_{\hat{\theta}_0 \in \mathbb{R}\times\{0\}} \sum_{k=1}^{2} (y(k) - \hat{\theta}_{0,k})^2 \\
&\quad - \min_{\hat{\theta}_1 \in \{0\}\times\mathbb{R}} \sum_{k=1}^{2} (y(k) - \hat{\theta}_{1,k})^2 \\
&= y(2)^2 - y(1)^2
\end{aligned}
$$

Example 2 : MI test

$$\mathcal{H}_0 : (\theta_1, \theta_2) \in \mathbb{R} \times \{0\}$$
$$\mathcal{H}_1 : (\theta_1, \theta_2) \in \mathbb{R} \times \mathbb{R}$$

apply maximally invariant test

generalized log likelihood over MI statistic

$$\begin{aligned} 2 \ln \hat{L}_R(t(\mathbf{y})) &= \max_{c \in \mathbb{R}} 2 \ln L\left(c; t(\mathbf{y})\right) - 2 \ln L\left(0; t(\mathbf{y})\right) \\ &= y(2)^2 - \min_{c \in \mathbb{R}} (y(2) - c)^2 \\ &= y(2)^2 \end{aligned}$$